Technologies and Services of Digital Broadcasting (10)

Coded Modulation Schemes

As described in previous issues, there are two main parts of digital transmission technology: digital modulation technology that converts a digital signal into a transmission signal, and error correction technology that overcomes errors caused by noise and/or distortion arising on a digital transmission path. These two fields have traditionally been researched independently. Coded modulation, on the other hand, can be viewed as an error correcting technology that integrates the above technologies giving particular consideration to non-uniform errors caused by multi-phase/multi-level modulation.

In multi-phase/multi-level modulation, the separation between signal points (symbols) in a constellation used for transmitting information may vary. Figure 1 shows these differences between symbol distances for various modulation schemes. As can be seen from the figure, the symbol distances of each modulation scheme are different. In QPSK, for example, the transmission of symbol S_0 is more likely to be erroneously received as S_1 or S_3 than as S_2. In 8PSK, diagonally placed symbols can maintain a distance of at least 2.6 times that of adjacent symbols.

Generally for coherent detection, the probability $p(x)$ of the error distance x between the received symbol and transmitted symbol is a Gaussian distribution expressed as follows.

$$p(x) = \frac{1}{2\pi N} e^{-\frac{x^2}{2N}} \qquad (1)$$

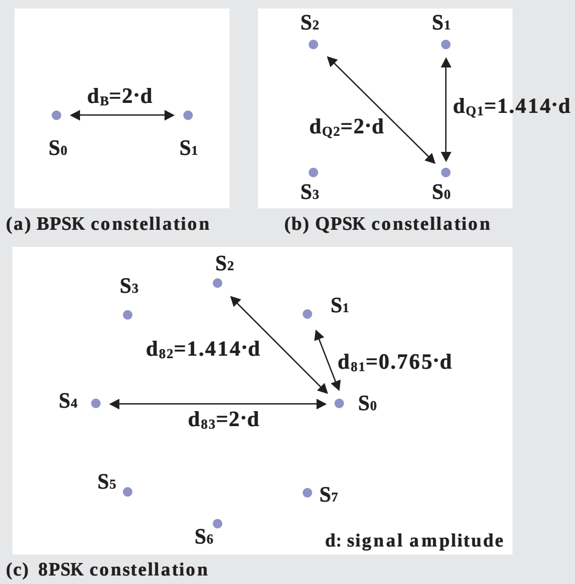


Figure 1: Difference in symbol distance

Here, N is noise variance. Figure 2 shows the probability density function for error in the distance between the received symbol and transmitted symbol. Denoting the distance between adjacent symbols as A, an error occurs when x becomes larger than $A/2$. Accordingly, the error probability P_e becomes

$$P_e = \int_{\frac{A}{2}}^{\infty} p(x)dx \qquad (2)$$

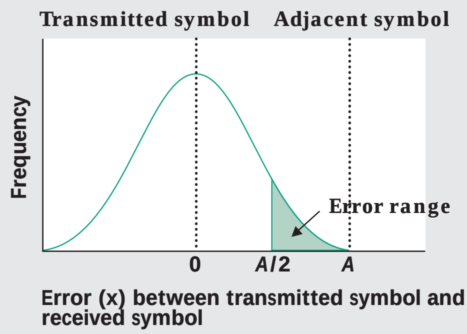


Figure 2: Probability density in distance between received symbol and transmitted symbol

Figure 3 shows P_e when normalizing the inter-symbol distance A by N. This plot tells us that the error rate with respect to inter-symbol distance decreases monotonically. For example, a normalized inter-symbol distance of 4 means an error rate of about 2×10^{-2}. For example, a normalized inter-symbol distance of 4 corresponds to an error rate of 2×10^{-2}, which is not conductive to digital transmission, while at or above 2.6 times this distance (10.6), the corresponding error rate is 10^{-7}. That means the difference of error rate is five orders of magnitude.

In light of the above, uniform error correction might not

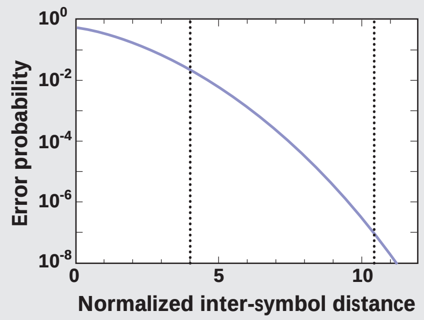


Figure 3: Error probability for normalized symbol distance

work effectively for a modulation scheme in which error occurrence is different depending on symbol distance. For this reason, the effectiveness of error correction that takes into account the symbol arrangement in a digital signal has come to be emphasized, beginning with Ungerboeck in 1976 and 1982 and Imai and Hirakawa in 1977. There has since been extensive research on coded modulation that integrates modulation and error correction.

1. Set Partitioning

A coded modulation scheme applies non-uniform error correction to non-uniform symbol distances for multi-phase/multi-level modulation. In other words, it applies error-correcting codes having different error-correcting capabilities. We recall here that multi-phase/multi-level modulation is characterized by the transmission of multiple bits per symbol. A specific technique of allocating bits when applying codes having different error-correcting capabilities is "set partitioning."

Figure 4 shows the bit allocation by set partitioning as proposed by Ungerboeck, taking 8PSK modulation as an example. This technique is as follows.

(1) Divide adjacent symbols on the 8PSK constellation into two groups, allocating 0 or 1 to the LSB (a_1) of each symbol, depending on the group.

(2) As each of these two groups consists of four symbols (corresponding to QPSK), divide adjacent symbols in each group in the same way as step (1), allocating 0 or 1 to the 2nd bit (a_2) of each symbol, depending on the group.

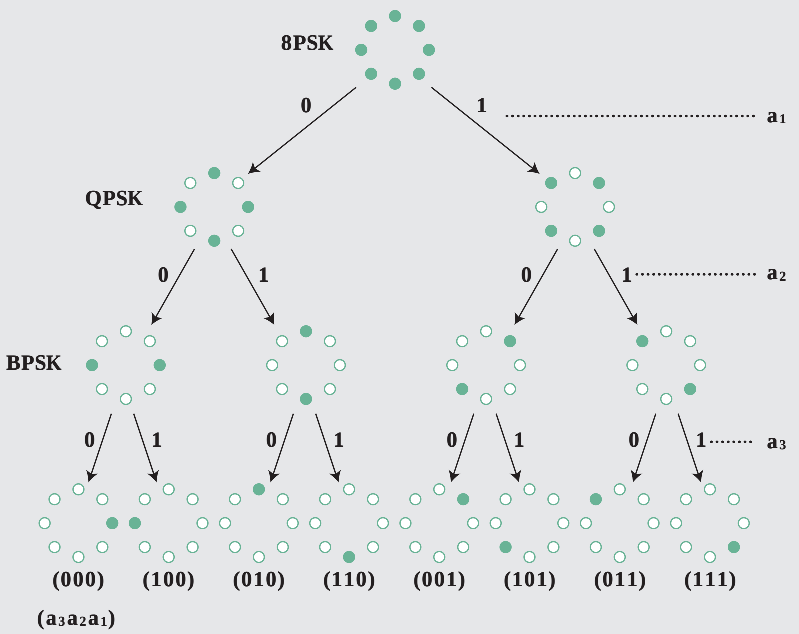
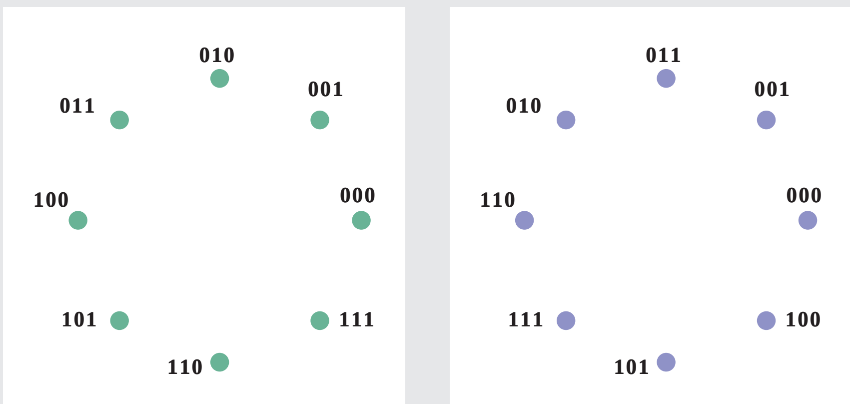


Figure 4: Set partitioning technique



(a) Bit allocation by set partitioning (a) Bit allocation by gray coding

Figure 5: 8PSK symbol arrangement charts

(3) As each of the resulting four groups consists of two symbols (corresponding to BPSK), again divide adjacent symbols in each group into two groups allocating 0 or 1 to the MSB (a_3) of each symbol, depending on the group.

Allocating bits by partitioning in this way reveals the following.

(1) Errors for bit a_1 can easily occur, because adjacent symbols of 8PSK will necessarily have different a_1s.

(2) If a_1 is assumed to be correct, then a_2 changes every other symbol of 8PSK and a symbol distance the same as that of QPSK will be obtained.

(3) If a_1 and a_2 are assumed to be correct, then a_3 can be determined if a decision can be made as to which diagonal symbol has been received, and a symbol distance the same as that of BPSK will be obtained.

This means that if a_1 can be guaranteed by using robust error-correcting code, characteristics equivalent to those of QPSK can be obtained, and if a_2 can be guaranteed, characteristics equivalent to those of BPSK can be obtained.

Bit allocation in the past has also made use of gray coding. Figure 5 shows the 8PSK symbol arrangement charts made by set partitioning and gray coding. Bit allocation using gray coding means that adjacent symbols differ by only one bit, that is, an arrangement in which the Hamming distance is 1. In this case, there is no difference in the ways that errors occur in bits a_1, a_2, and a_3.

Figure 6 shows the configuration of an 8PSK modulator using coded modulation, and Fig. 7 shows that of a conventional 8PSK modulator. Mapping A in Fig. 6 performs bit allocation by set partitioning. In this case, robust protection of bit a_1 is important, as described above. In the conventional modulator of Fig. 7, on the other hand, error correction code acts on each bit in a uniform manner, making it difficult to employ differences in correction capabilities. For this reason, bit allocation using gray coding provides better characteristics in Mapping B.

2. Coded Modulator and Its Characteristics

As shown by Fig. 6, bits in multi-phase/multi-level modulation may be transmitted using error-correcting codes of differing capability, which means that various schemes can be considered for coded modulation. One of these, proposed by Ungerboeck, exhibits superior characteristics through trellis-based decoding. Figure 8 shows the configuration of this scheme. In relation to Fig. 6, this scheme enables the use of coded modulation with 1/2 convolutional code for error-correcting codes 1 and 2.

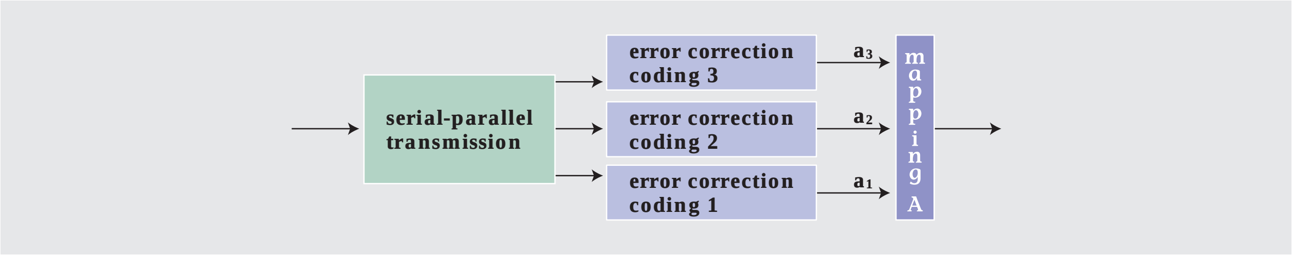


Figure 6: Configuration of an 8PSK modulator using coded modulation

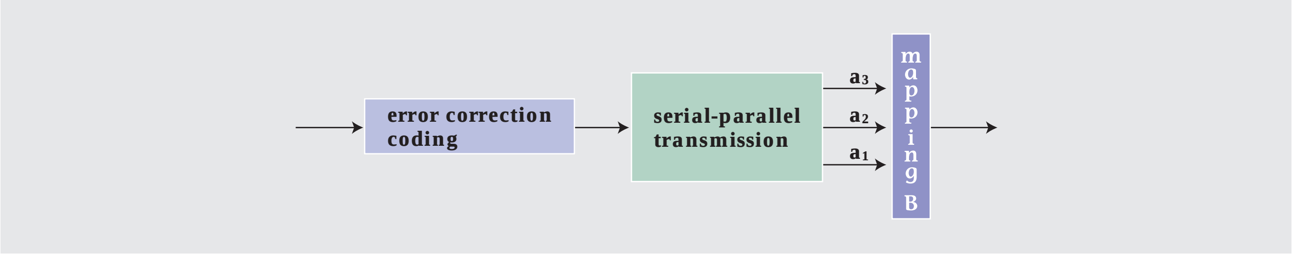


Figure 7: Configuration of a conventional 8PSK modulator

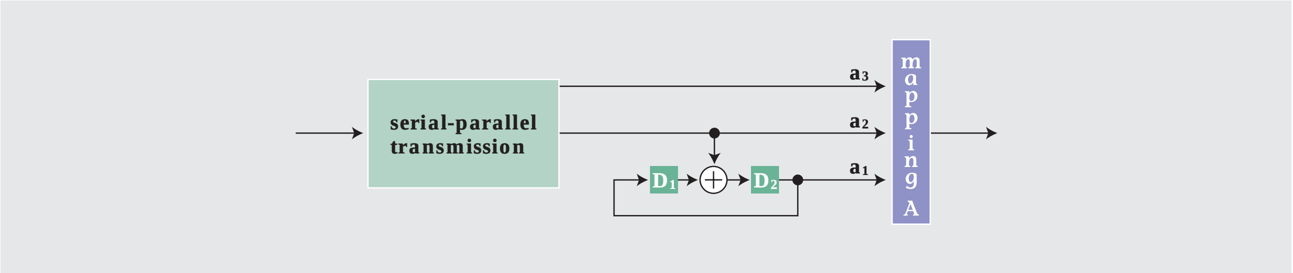


Figure 8: Configuration of an Ungerboeck coded modulator

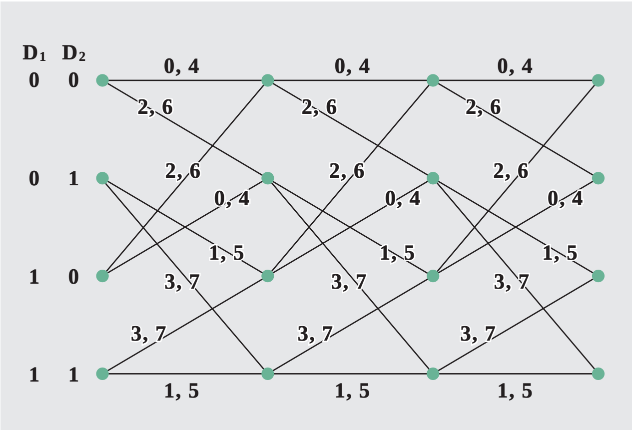


Figure 9: Decoding Trellis diagram for the Ungerboeck coded modulator

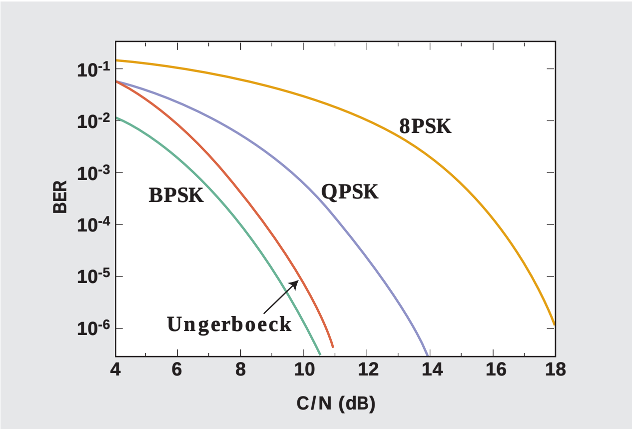


Figure 10: Performance of coded modulation using convolutional code

The use of convolutional code, in turn, makes possible Viterbi decoding using a Trellis diagram on the receive side. Figure 9 shows such a decoding trellis diagram. The numerals in the figure indicate symbols with (a_3, a_2, a_1) denoted in decimal. For example, 0=(0 0 0), 4=(1 0 0). As can be seen from Fig. 8, a_3 is not coded, which means that it cannot be distinguished in the decoding of error-correcting code. For this reason, two symbols with different a_3's are shown on the same line on the Trellis diagram, for example, (0,4) or (2,6). The metric or decoding criterion here is the square Euclidean distance from the received symbol to each symbol.

Figure 10 shows the characteristics of coded modulation using convolutional code. This plot reveals that the Ungerboeck scheme asymptotically approaches BPSK performance in the range for which 1/2 convolutional code can be sufficiently corrected. The transmission bit rate per Hertz of Ungerboeck coded modulation is the same as that of QPSK, which means that the transmission power for obtaining the same error rate as QPSK can be made smaller by 3 dB.

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